Chapter 8

Rise, Set, Twilight

General Visibility

For the planning of observations, it is useful to know the times during which a certain body is above the horizon as well as the times of sunrise, sunset, and twilight.

A body can be always above the horizon, always below the horizon, or above the horizon during a part of the day, depending on the observer's latitude and the declination of the body.

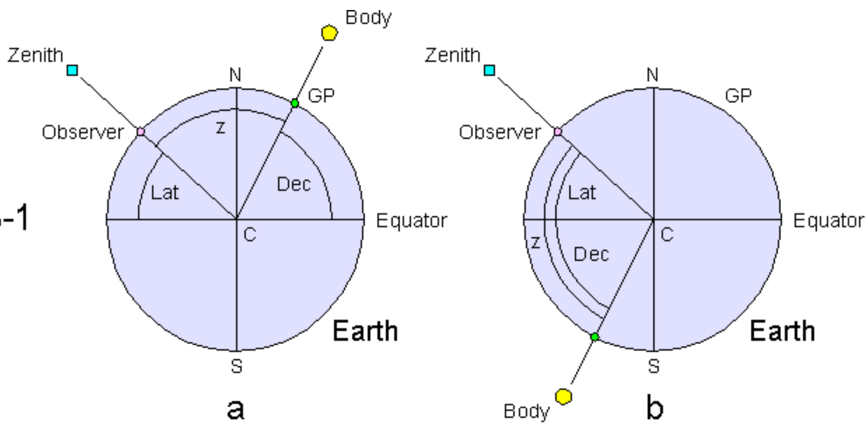
A body is **circumpolar** (always above the celestial horizon) if the zenith distance is smaller than 90° at the moment of **lower** meridian passage, i. e., when the body is on the lower branch of the local meridian (*Fig 8-1a*). This is the case under the following conditions:

$$Lat \cdot Dec > 0 \ \text{ AND } \ \left| Lat + Dec \right| > 90°$$

A body is continually below the celestial horizon if the zenith distance is greater than 90° at the instant of **upper** meridian passage (*Fig 8-1b*). The corresponding rule is:

$$Lat \cdot Dec < 0 \ \text{ AND } \ \left| Lat - Dec \right| > 90°$$

Fig. 8-1



A celestial body being on the same hemisphere as the observer is either sometimes above the horizon or circumpolar. A body being on the opposite hemisphere is either sometimes above the horizon or permanently invisible, but never circumpolar.

The sun provides a good example of how the visibility of a body is affected by latitude and declination. At the time of the summer solstice (Dec = +23.5°), the sun is circumpolar to an observer being north of the **arctic circle** (Lat > +66.5°). At the same time, the sun remains below the celestial horizon all day if the observer is south of the **antarctic circle** (Lat < –66.5°). At the times of the equinoxes (Dec = 0°), the sun is circumpolar only at the poles. At the time of the winter solstice (Dec = –23.5°), the sun is circumpolar south of the antarctic circle and invisible north of the arctic circle. If the observer is between the arctic and the antarctic circle, the sun is visible during a part of the day all year round.

Rise and Set

The events of rise and set can be used to determine latitude, longitude, or time. One should not expect very accurate results, however, since the atmospheric refraction may be erratic if the body is on or near the horizon.

The geometric rise or set of a body occurs when the center of the body passes through the celestial horizon (H = 0°). Due to the influence of atmospheric refraction, all bodies except the moon appear above the visible and sensible horizon at this instant. The moon is not visible at the moment of her geometric rise or set since the depressing effect of the horizontal parallax (~1°) is greater than the elevating effect of atmospheric refraction.

The approximate apparent altitudes (referring to the sensible horizon) at the moment of the astronomical rise or set are:

Sun (lower limb): 15'

Stars: 29'

Planets: 29' – HP

When measuring these altitudes with reference to the sea horizon, we have to add the dip of horizon (chapter 2) to the above values. For example, the altitude of the lower limb of the rising or setting sun is approx. 20' if the height of eye is 8m.

We begin with the well-known altitude formula (see chapter 4).

$$\sin H \ = \ 0 \ = \ \sin Lat \cdot \sin Dec + \cos Lat \cdot \cos Dec \cdot \cos \ t$$

$$\cos \ t \ = \ -\frac{\sin Lat \cdot \sin Dec}{\cos Lat \cdot \cos Dec}$$

Solving the equation for the meridian angle, t, we get :

$$t \ = \ \arccos \ \left(-\tan Lat \cdot \tan Dec \right)$$

The equation has no solution if the argument of the inverse cosine is smaller than –1 or greater than 1. In the first case, the body is **circumpolar**, in the latter case, the body remains continuously below the horizon. Otherwise, the arccos function returns values in the range from 0° through 180°.

Due to the ambiguity of the arccos function, the equation has two solutions, one for rise and one for set. For the calculations below, we have to observe the following rules:

If the body is **rising** (body eastward from the observer), t is treated as a **negative** quantity.

If the body is **setting** (body westward from the observer), t is treated as a **positive** quantity.

If we know our latitude and the time of rise or set, we can calculate our longitude:

$$Lon \ = \ \pm t - GHA$$

GHA is the Greenwich hour angle of the body at the moment of rise or set. The sign of t has to be observed carefully (see above). If the resulting longitude is smaller than –180°, we add 360°.

Knowing our position, we can calculate the times of sunrise and sunset:

$$GMT_{Surise/set} \ = 12 \ \pm \ \frac{t[\circ]}{15} - \frac{Lon[\circ]}{15} - EoT$$

The times of sunrise and sunset obtained with the above formula are not quite accurate since Dec and EoT are variable. Since we do not know the exact time of rise or set at the beginning, we have to use estimated values for Dec and EoT initially. The time of rise or set can be improved by iteration (repeating the calculations with Dec and EoT at the calculated time of rise or set). Further, the times thus calculated are influenced by the irregularities of atmospheric refraction near the horizon. Therefore, a time error of ±2 minutes is not unusual.

Accordingly, we can calculate our longitude from the time of sunrise or sunset if we know our latitude:

$$Lon\left[\circ\right] = \pm t + 15 \cdot \left(12 - GMT_{Sunrise/set} - EoT\right)$$

Again, this is not a very precise method, and an error of several arcminutes in longitude is not unlikely.

Knowing our longitude, we are able to determine our approximate latitude from the time of sunrise or sunset:

$$t\left[\circ\right] = Lon\left[\circ\right] - 15 \cdot \left(12 - GMT_{Sunrise/set} - EoT\right)$$

$$Lat = \arctan\left(-\frac{\cos t}{\tan Dec}\right)$$

In navigation, rise and set are defined as the moments when the upper limb of a body is on the visible horizon. These events can be observed without a sextant. Now, we have to take into account the effects of refraction, horizontal parallax, dip, and semidiameter. These quantities determine the altitude (Ho) of a body with respect to the celestial horizon at the instant of the visible rise or set.

$$t = \arccos\frac{\sin Ho - \sin Lat \cdot \sin Dec}{\cos Lat \cdot \cos Dec}$$

$$Ho = HP - SD - R_H - Dip$$

According to the Nautical Almanac, the refraction for a body being on the sensible horizon, R_H, is approximately (!) 34'.

When observing the upper limb of the sun, we get:

$$Ho = 0.15' - 16' - 34' - Dip \approx -50' - Dip$$

Ho is negative. If we refer to the upper limb of the sun and the sensible horizon (Dip=0), the meridian angle at the time of sunrise or sunset is:

$$t = \arccos\frac{-0.0145 - \sin Lat \cdot \sin Dec}{\cos Lat \cdot \cos Dec}$$

Azimuth and Amplitude

The azimuth angle of a rising or setting body is calculated with the azimuth formula (see chapter 4):

$$Az = \arccos\frac{\sin Dec - \sin H \cdot \sin Lat}{\cos H \cdot \cos Lat}$$

With H=0, we get:

$$Az = \arccos \frac{\sin Dec}{\cos Lat}$$

Az is +90° (rise) and –90° (set) if the declination of the body is zero, regardless of the observer's latitude. Accordingly, the sun rises in the east and sets in the west at the times of the equinoxes (geometric rise and set).

With H_{center}= –50' (upper limb of the sun on the sensible horizon), we have:

$$Az = \arccos \frac{\sin Dec + 0.0145 \cdot \sin Lat}{0.9999 \cdot \cos Lat}$$

The true azimuth of the rising or setting body is:

$$Az_N = \begin{cases} Az & \text{if} \quad t < 0 \\ 360° - Az & \text{if} \quad t > 0 \end{cases}$$

The azimuth of a body at the moment of rise or set can be used to find the magnetic declination at the observer's position (compare with chapter 13).

The horizontal angular distance of the rising/setting body from the east/west point on the horizon is called **amplitude** and can be calculated from the azimuth. An amplitude of E45°N, for instance, means that the body rises 45° north of the east point on the horizon.

Twilight

At sea, twilight is important for the observation of stars and planets since it is the only time when these bodies **and** the horizon are visible. By definition, there are three kinds of twilight. The altitude, H, refers to the center of the sun and the celestial horizon and marks the beginning (morning) and the end (evening) of the respective twilight.

Civil twilight: H = –6°

Nautical twilight: H = –12°

Astronomical twilight: H = –18°

In general, an altitude of the sun between –3° and –9° is recommended for astronomical observations at sea (best visibility of brighter stars and sea horizon). However, exceptions to this rule are possible, depending on the actual weather conditions.

The meridian angle for the sun at –6° altitude (center) is:

$$t = \arccos \frac{-0.10453 - \sin Lat \cdot \sin Dec}{\cos Lat \cdot \cos Dec}$$

Using this formula, we can find the approximate time for our observations (in analogy to sunrise and sunset).

As mentioned above, the simultaneous observation of stars **and** the horizon is possible during a limited time interval only.

To calculate the length of this interval, ΔT, we use the altitude formula and differentiate sin H with respect to the meridian angle, t:

$$\frac{d\left(\sin H\right)}{d\,t} = -\cos Lat \cdot \cos Dec \cdot \sin t$$

$$d\left(\sin H\right) = -\cos Lat \cdot \cos Dec \cdot \sin t \cdot d\,t$$

Substituting cosH·dH for d(sinH) and solving for dt, we get the change in the meridian angle, dt, as a function of a change in altitude, dH:

$$d\,t = -\frac{\cos H}{\cos Lat \cdot \cos Dec \cdot \sin t} \cdot d\,H$$

With H = –6° and dH = 6° (H = –3°...–9°), we get:

$$\Delta\,t\left[\circ\right] \approx -\frac{5.97}{\cos Lat \cdot \cos Dec \cdot \sin t}$$

Converting the change in the meridian angle to a time span (measured in minutes) and ignoring the sign, the equation is stated as:

$$\Delta\,T\left[m\right] \approx \frac{24}{\cos Lat \cdot \cos Dec \cdot \sin t}$$

The shortest possible time interval for our observations (Lat = 0, Dec = 0, t = 96°) lasts approx. 24 minutes. As the observer moves northward or southward from the equator, cos Lat and sin t decrease (t>90°). Accordingly, the duration of twilight increases. When t is 0° or 180°, ΔT is infinite.

This is in accordance with the well-known fact that twilight is shortest in equatorial regions and longest in polar regions.

We would obtain the same result when calculating t for H = –3° and H = –9°, respectively:

$$\Delta\,T\left[m\right] = 4 \cdot \left(t_{-9^\circ}\left[\circ\right] - t_{-3^\circ}\left[\circ\right]\right)$$

The Nautical Almanac provides tabulated values for the times of sunrise, sunset, civil twilight and nautical twilight for latitudes between –60° and +72° (referring to an observer being at the Greenwich meridian). In addition, times of moonrise and moonset are given.